Number theory

GCD, LCM, Divisibility

Greatest common divisor: GCD(a, b) is largest number that divides both a and b

- ➔ Ex: GCD(15, 20) = 5
- ➔ GCD(1, 15)?
- ➔ GCD(3, 9)?

Relatively prime numbers share no common factors, if a, b are relatively prime GCD(a, b) = 1

LCM: Least common multiple

LCM(a, b) is smallest number that is a multiple of both a and b

- ➔ Ex: LCM(15, 20) = 60
- ➔ LCM(1, 15)?
- ➔ LCM(3, 9)?
- ➔ LCM(8, 6)?

The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c?

$\textbf{(A)}\ 20 \qquad \textbf{(B)}\ 30 \qquad \textbf{(C)}\ 60 \qquad \textbf{(D)}\ 120 \qquad \textbf{(E)}\ 180$

Prime factorization: write number as a product of primes

- ➔ $60 = 2^2 \cdot 3 \cdot 5$
- ➔ What is prime factorization of 75?

What is the sum of the prime factors of 2010?

$\textbf{(A)}\ 67 \qquad \textbf{(B)}\ 75 \qquad \textbf{(C)}\ 77 \qquad \textbf{(D)}\ 201 \qquad \textbf{(E)}\ 210$

Find GCD, LCM with prime factorization.

Write prime factorization

GCD: take minimum of exponents

LCM: take maximum of exponents

> What is the ratio of the least common multiple of 180 and 594 to the greatest common factor of 180 and 594?
> $\textbf{(A)}\ 110 \qquad \textbf{(B)}\ 165 \qquad \textbf{(C)}\ 330 \qquad \textbf{(D)}\ 625 \qquad \textbf{(E)}\ 660$

Additional result: GCD(a, b) * LCM(a, b) = ab

How many ordered pairs (a, b) of positive integers satisfy the equation

$$a \cdot b + 63 = 20 \cdot \text{lcm}(a, b) + 12 \cdot \gcd(a, b),$$

where $\gcd(a, b)$ denotes the greatest common divisor of a and b, and $\text{lcm}(a, b)$ denotes their least common multiple?

$\textbf{(A)}\ 0 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 4 \qquad \textbf{(D)}\ 6 \qquad \textbf{(E)}\ 8$

Divisibility by 3: sum of digits is divisible by 3

Divisibility by 9: sum of digits is divisible by 9

What are the values of a, b such that 123a23b1 is as large as possible and divisible by 9?

Divisibility by 4: as long as last two digits are divisible by 4, whole number is divisible by 4

Problem 24

The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS is divisible by 5, and the three-digit number RST is divisible by 3. What is P?

$\textbf{(A)}\ 1 \qquad \textbf{(B)}\ 2 \qquad \textbf{(C)}\ 3 \qquad \textbf{(D)}\ 4 \qquad \textbf{(E)}\ 5$