

Class 1: Introduction

Madam Math Circle 6/7

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Introduction

The math we're going to do is different from school math. Think of these problems as a puzzle you try to figure out. Don't expect to get every question right, just enjoy the process of figuring out the problems and understanding the solution.

Today, we'll be doing problems that use concepts that you've already learned in normal school but to solve these problems you need to apply them in a more creative way.

- How many numbers are there between 5 and 25, inclusive?

While it might seem that the answer is $25 - 5 = 20$, this is actually incorrect. The correct answer is actually 21. One way to see this is to subtract 4 from both 5 and 25, giving 1 to 21, which obviously contains 21 numbers. In general, there are $a - b + 1$ numbers between b and a inclusive.

- How many multiples of 6 are there between 13 and 57?

We can first start by finding the smallest and largest multiple of 6 in the interval. The smallest is $6 * 3 = 18$ and the largest is $6 * 9 = 54$. Thus, we know that 6 times any number from 3 to 9 is a multiple of 6 in that interval. As a result, there are $9 - 3 + 1 = 7$ multiples of 6 between 13 and 57.

If you struggle with this, it is a good idea when doing these sorts of things to try to translate the sequence you are counting to $1, 2, \dots$. For example, if you are working with $18, 24, \dots, 54$, you can divide every number by 6, giving you $3, 4, \dots, 9$. You can then subtract every number by 2, giving you $1, 2, \dots, 7$.

- (2018 AMC 8 #5) What is the value of $1 + 3 + 5 + \dots + 2017 + 2019 - 2 - 4 - 6 - \dots - 2016 - 2018$?

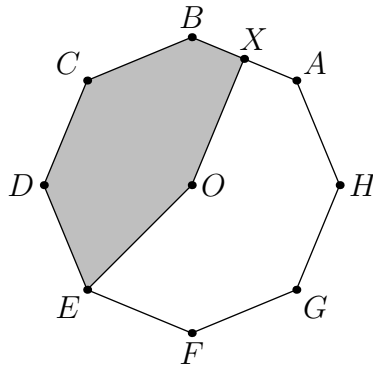
We can group this as $(1 - 2) + (3 - 2) + \dots + (2017 - 2018) + 2019$, which equals $1009 \cdot -1 + 2019 =$
2010

Guided Practice

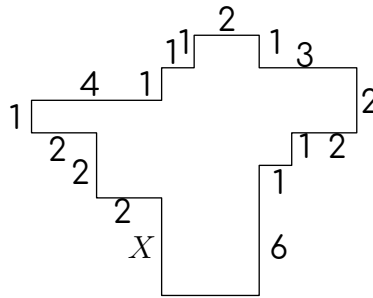
1. (2014 AMC 8 #2) Paul owes Paula 35 cents and has a pocket full of 5-cent coins, 10-cent coins, and 25-cent coins that he can use to pay her. What is the difference between the largest and the smallest number of coins he can use to pay her?
2. (2013 AMC 8 #3) What is the value of $4 \cdot (-1 + 2 - 3 + 4 - 5 + 6 - 7 + \dots + 1000)$?
3. (2018 AMC 8 #2) What is the value of the product

$$\left(1 + \frac{1}{1}\right) \cdot \left(1 + \frac{1}{2}\right) \cdot \left(1 + \frac{1}{3}\right) \cdot \left(1 + \frac{1}{4}\right) \cdot \left(1 + \frac{1}{5}\right) \cdot \left(1 + \frac{1}{6}\right)?$$

4. (2015 AMC 8 #2) Point O is the center of the regular octagon $ABCDEFGH$, and X is the midpoint of the side \overline{AB} . What fraction of the area of the octagon is shaded?



5. (2012 AMC 8 #5) In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is X in centimeters?

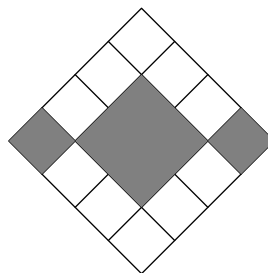


6. (MOEMS Vol 2 Set 7E 2C) Find the 4-digit number $ABCD$ if

$$\begin{array}{r} ABCD \\ \times \quad 9 \\ \hline ABCD \end{array}$$

Homework

- (2016 AMC 8 #1) The longest professional tennis match ever played lasted a total of 11 hours and 5 minutes. How many minutes was this?
- (MOEMS Vol 2 Set 7E 2D) How many multiples of 7 are there between 100 and 1000?
- There are 20 girls at a party. 15 of them like chocolate ice cream and 10 of them like whipped cream. How many girls like both chocolate ice cream and whipped cream?
- (2010 AMC 8 #2) If $a@b = \frac{a \times b}{a+b}$ for a, b positive integers, then what is $5@10$?
- (2008 AMC 8 #6) In the figure, what is the ratio of the area of the gray squares to the area of the white squares?



6. (2007 AMC 8 #9) To complete the grid below, each of the digits 1 through 4 must occur once in each row and once in each column. What number will occupy the lower right-hand square?

1		2	
2	3		
			4